COMPUTER-ORIENTED STATISTICAL METHODS FOR LOW FAILURE PROBABILITY FATIGUE LIFE PREDICTION AND IMPACT STRENGTH LOWER BOUND DETERMINATION

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ABSTRACT

Safety and/or economic reasons point to the necessity of using lower-bounded statistical distribution functions for smoothing and extrapolating fatigue life experimental data. Practical experience with Weibull distributions is satisfactory to a certain degree, but problems exist in calculation of the confidence limits and proof of the convergence postulate. It is therefore considered necessary to experiment with more versatile discrete statistical distributions.

A proposed unified procedure for the evaluation of experimental data reduces the number of tests which must be carried out. Similarly, cross-plotting for Wohler (S-N) curve smoothing may be extended to include all failure probability levels. The same basic procedures may also be applied to lower-bound extrapolation of Charpy impact tests.

NOTATION

a	slope of Weibull distribution
b, b_1	Weibull constants
j	number of test specimens
r	correlation coefficient
n	number of load cycles applied
Δr_w	weighted inaccuracy number
w	impact energy
x	experimental parameter
D^*	'zero' fatigue damage
HB	Brinell hardness number
N	number of load cycles to failure
N_0	number of load cycles for zero failure probability
N_{50}	number of load cycles for 50% failure probability
P	failure probability
σ	normal stress
α	experimental parameter exponent
ξ	load cycle number (non-dimensional)

EDITOR'S NOTE:

The text of this paper is given in the edited and abbreviated form in which it was presented at the 10th Congress of the International Council of the Aeronautical Sciences.

The editor wishes to apologize to the author for any errors in presentation which may have arisen.

Copies of the original paper may be obtained upon application to the author.

Subscripts

a	denotes tests conducted at first stress level
b	denotes tests conducted at second stress level

INTRODUCTION

Fatigue failure in airframe components may be classified broadly in two groups: those which directly affect flight safety, and those which have economic consequences only. In the former case, a reasonably practical protection against fatigue failure may be achieved by designing for a failure probability of the order of 10^{-5}. In the latter case, the most economical service life may correspond to a nominal failure probability of the order of 0.1 percent. In both cases, there is a real problem because of the inherently wide scatter in fatigue test results.

Mean value and deviation theorems being the most common and most reliable ones used in mathematical statistics, early fatigue test evaluations relied heavily upon them. In practice, this has resulted in the use of log-normal or of two-parameter Weibull distributions. Provision against the possibility of service failure was achieved by the use of a sufficiently low nominal failure probability.

In theory as well as in practice, the expression of a really safe service life is synonymous with the corresponding fatigue life distributions converging on some type of lower-bounded statistical distribution function, except, of course, in the case of fail-safe structures.

Mean values and standard deviations are always within the limits of the experimentally measured fatigue life values, needing some kind of interpolation procedure for their calculation. Opposed to this, lower bounds are by their very nature outside the experimental data range, necessitating some form of extrapolation. It is at this juncture that the use of a computer is practically mandatory, as all useful statistical extrapolation procedures known to the author require extensive calculation.

SINGLE-LEVEL FATIGUE TESTS

Safe-Life Ratio

Before discussing in detail lower-bound extrapolation, let us examine briefly the possibilities of circumventing this problem. Cost-conscious industry managers frequently demand the determination of an allowable service life on the basis of two or three tests. Efforts in this direction always assume that the true safe service life may be not less than some convenient percentage of the mean or median life as determined from tests. Figure 1 shows some of our experimental data which strongly contradicts this assumption.

Nominal safe-life ratios, as determined by a procedure which will be introduced later, do not appear to have guaranteed minima. For median lives not exceeding 60,000 cycles, we did not achieve any positive safe life at all, and for longer median lives (that is, at lower stress ranges), the safe life ratio is critically influenced by detail design and the quality of workmanship. We have had cases of welding faults lowering the median life to 15 percent, resulting in near-zero safe life values.

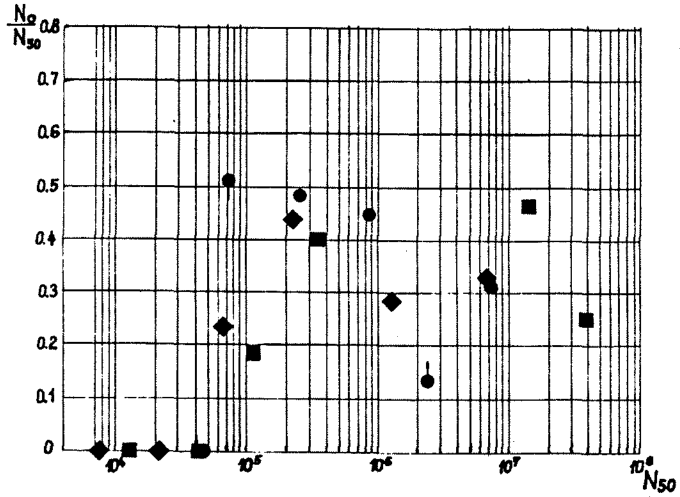

FIG. 1: NOMINAL SAFE LIFE RATIOS AS FUNCTION OF THE MEDIAN LIFE

Weibull Statistics

In view of the preceding remarks, individual lower bound determination for all test series is to be recommended. For this type of work, use of the so-called three-parameter Weibull distribution (known simply as a Weibull distribution) is customary. The failure probability function of this distribution is:

$$P(N) = 1 - 10^{b_1 (N - N_0)^a} = W(N) \qquad (1)$$

By double differentiation this becomes:

$$\log \log \frac{1}{1 - P(N)} = a. \log(N - N_0) + b \qquad (2)$$

Weibull analysis may be carried out numerically by the method of least-squares or by spline-fit equations, but most authors (Refs. 1 and 2) recommend graphical solution. The weakness of this is that for the usual fatigue life series, there is no value of N_0 which gives an exact straight-line fit. There is, therefore, a substantial uncertainty in estimation of the safe life. This problem may be solved by looking for the maximum correlation coefficient of the straightened Weibull plot. Our single-stress level fatigue test evaluation program which we call WEIBULL F (Ref. 3) uses this approach — see Figure 2. It may be regarded as an arithmetic counterpart of the graphical method, and gives practically identical results with those obtained by the use of spline-fit equations.

Although our experiences with the Weibull distribution are quite good so far, there are nevertheless some indications of possible problem areas.

Confidence Limits and Convergence

The statement that Weibull parameters as determined from two test specimens may not coincide with those determined from a much larger population sample does not need any formal proof. Unfortunately, there does not yet exist any universally-accepted standardized Weibull confidence limit calculation procedure. The common basic assumption is that if the number of tests could be increased to infinity, then the experimental life distribution would correspond with the Weibull distribution. Calculation of confidence limits is based on this assumption, using usual probability theory methods.

However, there are mathematical difficulties in execution of the calculations for the full three-parameter Weibull function. Johnson (Ref. 1) uses the two-parameter variant, as do several other authors. Amstadter (Ref. 2) endeavours to evade the problem on the tacit assumption that the calculated values of N_0 are correct. In our own country, Marialigeti (Ref. 4) has recently published a three-parameter confidence limit calculation method assuming the correctness of the Weibull slope as determined by experiment.

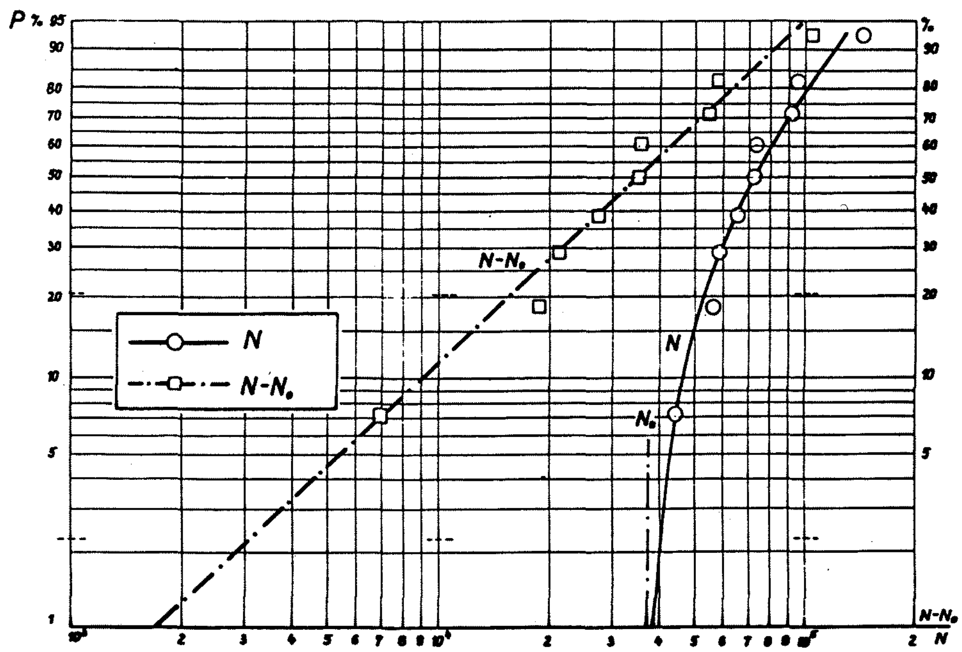

FIG. 2: WEIBULL GRAPH OF FATIGUE TEST SERIES AND DETERMINATION OF BEST FIT N_0

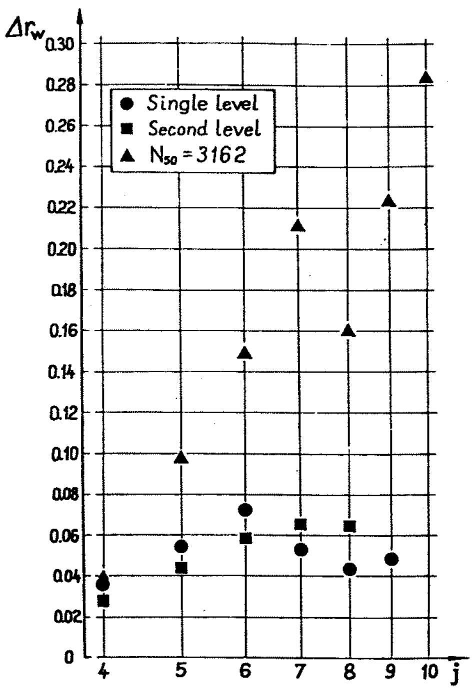

FIG. 3: WEIGHED INACCURACY NUMBERS FOR WEIBULL PLOTS OF FATIGUE TESTS

Quite distinct from the mathematical difficulties referred to, there is a yet more fundamental problem — that is, verification of the convergence postulate. To the best knowledge of the author, this has not yet been proved, neither in the single nor in the general case. We have attempted to obtain some data which may lead towards a solution of this problem by use of the so-called progressive data acquisition simulation process, which is one of the options of our fatigue evaluation program. Calculation of the Weibull parameters proceeds as as follows:

Experimental data are fed into the computer in the order in which the tests were carried out. The program takes the first four results and grades them according to the fatigue lives obtained. Best-fit Weibull parameters are computed and printed out, together with the values of N_{50}, the correlation coefficient and the weighted inaccuracy number given by:

$$\Delta r_w = (1 - r) \sqrt{j - 3} \qquad (3)$$

Following this, the fifth result is taken, graded, and the best-fit Weibull parameters are re-calculated. This process is repeated until all test results have been evaluated.

During progressive data acquisition simulation, the values of the correlation coefficient may oscillate in a random manner, but they must exhibit a steadily-improving trend. If the convergence postulate is true, then this trend has to be sufficiently strong that the sequence of the weighted inaccuracy numbers does not exhibit an increasing trend. The best way to check on this is by calculating the means of several test series on similar test specimens. Results of some calculations of this nature are shown on Figure 3.

Data for the series marked by circles and squares were taken from tests conducted on a Dural-type alloy (Ref. 5). The circles indicate mean values for a series of eight single-level tests, while the squares indicate the means for a series of 27 two-level tests. In the latter case, the Weibull evaluation has been carried out for the number of cycles to failure at the second stress level. In both cases, the weighted inaccuracy numbers increase from j=4 to j=6, indicating what might be termed uncertainty trends. On the whole, we would have to say that there are insufficient data to give significant trends except for the evidence presented by the third series of tests, the data for which are marked by triangles. These were single-level tests carried out at a stress level giving a median life in excess of 3000 cycles. In this particular case there is an increase of nearly one order of magnitude in the value of the weighted inaccuracy number, indicating a distinctly non-Weibullian characteristic for the life distribution. Our experience so far indicates that such behaviour is common in all cases approximating low-cycle fatigue.

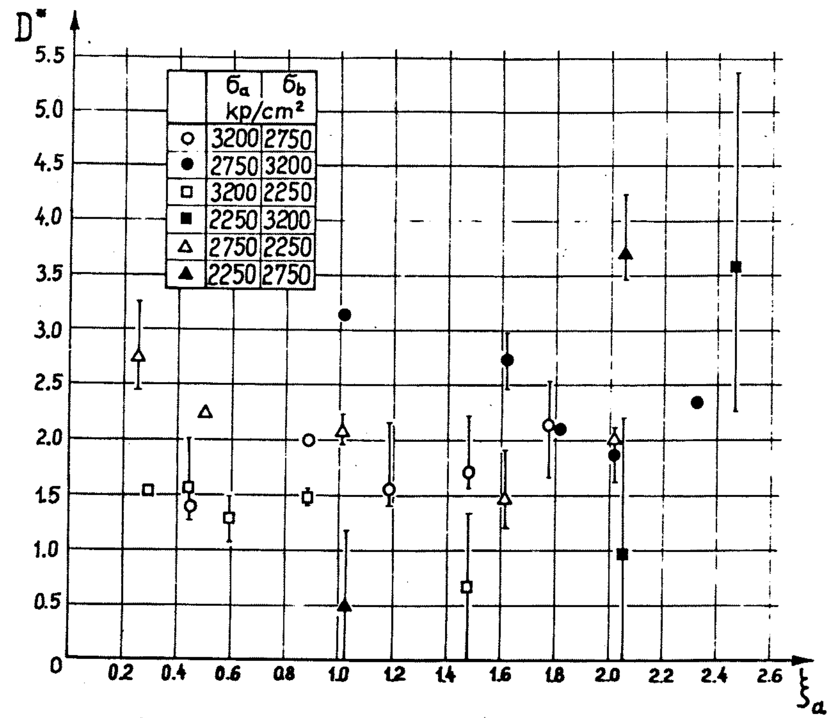

FIG. 4: "ZERO" TWO-LEVEL FATIGUE DAMAGE OF DURAL ROTATING-BENDING SPECIMENS

This duality may be resolved in two ways. One either accepts the fact that there is a different fatigue process in those cases in which the stress level is such as to give median lives less than N_{50} equal to approximately 50,000 cycles, or one postulates the existence of a non-Weibullian fatigue life distribution which approximates the Weibull distribution at lower stress levels but has a different shape at higher stress levels. The author prefers the latter view.

TWO-LEVEL FATIGUE TESTS

Nominal Safe Life

Two-level fatigue tests are most useful research tools for investigating the problems of cumulative fatigue damage. For 50 percent failure probability, the fatigue lives actually obtained were less than those predicted by Miner's rule (Ref. 6) when the higher stress was applied first, and greater when the lower stress was applied first. At this level of experimentation, not much more information is to be expected. Fortunately, zero failure probability fatigue life trends appear to have more regular characteristics. Development of our fatigue life evaluation program since 1970 has led us to revise our former calculations on two-level rotating bending fatigue tests on Dural specimens. Figure 4 shows some of the results obtained. The cumulative damage at the zero failure level based on Miner's rule has been calculated by the following formula:

$$D^* = \frac{n_a}{(N_0)_{\sigma_a}} + \frac{N_0 b}{(N_0)_{\sigma_b}} = \xi_a + \xi_b \tag{4}$$

where D* is the Miner damage at zero failure level.

Our current N_0 values are probably more accurate than those obtained in our earlier work in 1970, and the results are plotted on Figure 4. The plain circles are for those results in which the higher stress was applied first, while the filled-in circles are for those results in which the low stress level was applied first.

PRACTICAL CONCLUSIONS

In the preceding substance of this paper, we have spoken of difficulties in:

(a) Calculation of three-parameter Weibull confidence limits.

(b) Proof of the convergence of the experimental fatigue life sequences to the Weibull distribution.

(c) Calculation of safe life for non-uniform stress levels.

Other problems also arise, such as:

(d) Ambiguity in median rank ranging.

(e) Lack of an upper bound to the Weibull distribution.

There are two ways in which we are trying to improve the current situation. Firstly, we have developed what we call a discrete probability procedure for the three-parameter Weibull confidence limit calculations. As part of our fatigue evaluation program, we have been accumulating practical experience over a period of several months in order to assess the accuracy of our procedure. Secondly, we plan to substitute a family of discrete probability distributions in place of the usual Weibull distribution. In this latter case, we hope to obtain better agreement between experimental results and theory. Subject to the usual delays to be expected in the development of a new computer program, we hope that we shall be ready to proceed with preliminary trials at the end of this year.

EXPERIMENTAL DEVELOPMENT

The fatigue endurance of airframe components is critically dependent not only upon the stress level and on detail design, but also on the quality and accuracy of manufacturing processes. This is why a substantial percentage of fatigue tests is carried out in order to determine the best combination of the various parameters.

Comparison of two or more totally different technologies can be made by calculating the confidence number for the experimental life ratios at the required failure probability level (Refs. 1, 2 and 7). The same methods may be applied if compliance with the prescribed technology is to be controlled, as would be the case in a production contract. The problem is far more complicated when one has to take into consideration the optimum value or allowable limits of tempering hardness, alloy content, surface finish, and numerous other variables.

The usual approach to problems of this nature is to run a series of constant-parameter tests on a number of different specimens.

After calculation of the best-fit life distributions for each, fatigue lives for an appropriate failure probability level can be plotted as a function of the parameter value. Optimum parameter values and limits can be read from graphs. However, this procedure is not very satisfactory because of the large number of tests required, resulting in high costs and considerable time. We hope that we have approached a solution to this problem in the development of an approximate evaluation procedure which requires no constant-parameter series, but only knowledge of the parameter value for each individual test specimen.

Mathematical Basis of the Unified Evaluation Procedure

Let the numerical value of the parameter under test be denoted by x. The different fatigue endurance limits of the individual test specimens are characterized in the normal single-level test evaluation procedure by the failure probability P. For a manufacturing process of acceptable quality the individual fatigue lives as observed by test may be regarded as a sample series taken from the infinite life distribution:

$$N = N(x, P) \tag{5}$$

This unknown distribution may be expressed in the form:

$$P = P(x, N) \tag{6}$$

If the form of the life distribution function is not dependent upon x, then we can substitute for this in the first approximation as:

$$P \cong P(f(x) \cdot N) = W(f(x) \cdot N) \tag{7}$$

provided, of course, that we are using a Weibull distribution.

The final proof of the correctness of this assumption can be gained only by practical experience.

The computation sequence proceeds as follows:

Firstly, it is advisable to plot cycles to failure as a function of x for all test specimens. From this plot, an estimate is made of a suitable function type denoted by f(x). The next step is regression analysis for the f(x) parameters. This is carried out by the computer program. The procedure is outlined in the following example.

Numerical Example

The example chosen is from a series of tests on leaf springs carried out as part of a heat-treatment research program conducted at the Budapest Technical University (Ref. 8). Series 31, 32 and 34 of these were selected in order to test the evaluation procedure (Ref. 9). The Brinell hardness was selected as a numerical parameter for the strength of the material after heat treatment. Individual Brinell hardness and fatigue lives are plotted on Figure 5. Data for series 31 are indicated by squares, series 32 by triangles, and series 34 by circles.

For the sake of simplicity, the following expression was used as a weighting function:

$$f(x) = x^{\alpha} \quad (8)$$

which yields the unified failure probability function:

$$P \cong W(x^{\alpha} \cdot N) \quad (9)$$

Regression analysis and subsequent search gave a value of $\alpha = -1.124$ as an optimum fit exponent, resulting in the distribution indicated by the solid line on the right-hand side of Figure 6. The computed nominal safe life as a function of Brinell hardness is indicated by the solid line drawn on Figure 5.

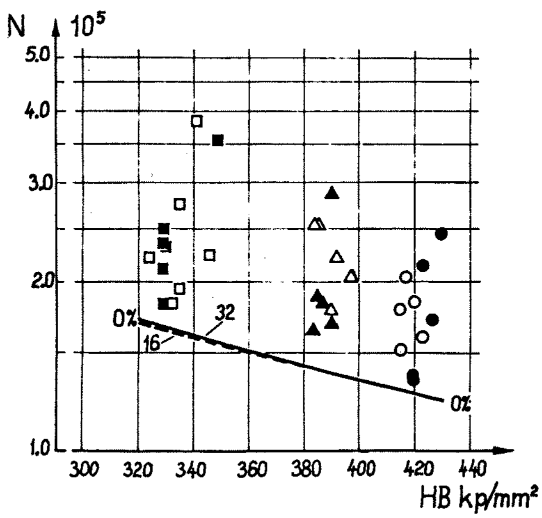

FIG. 5: THE INFLUENCE OF TEMPERING HARNESS ON THE FATIGUE LIFE OF SPRING STEEL LEAVES

We also checked to see if it was possible to use fewer test specimens without impairing the reliability of the results. The unified evaluation procedure was repeated using only the first half of the test results. The value of the optimum fit exponent was calculated at -1.051, and the resulting unified distribution is shown by the dotted line on Figure 6. At first sight the difference appears to be considerable, but the plot of Brinell hardness against fatigue life (the dotted line on Figure 5) and Table I indicates that the nominal safe lives (zero failure probability) are practically unchanged.

While the simple power law weighting function may be satisfactory for a number of cases characterized by a monotonic increase or decrease in fatigue life for increasing parameter values, local maxima or minima may necessitate the use of different function types. Nevertheless, this does not change the essence of the method.

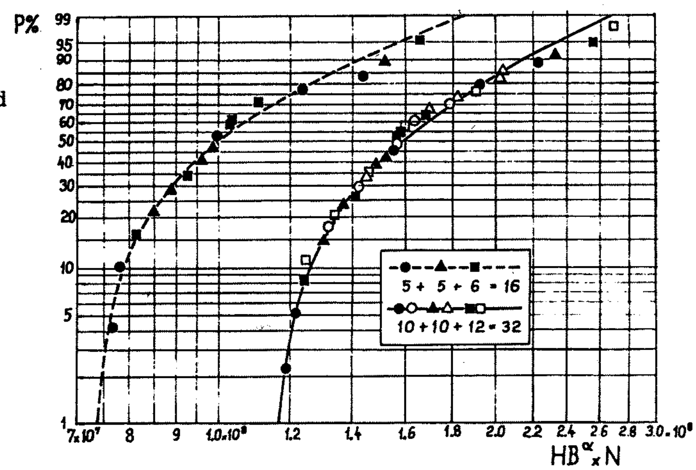

FIG. 6: UNITED WEIBULL GRAPH OF THE SPRING LEAVE FATIGUE TESTS

Ser. No.	HB kp/mm²	N_0 as from Eq.(9) 32 pieces	N_0 as from Eq.(9) 16 pieces	N_0 from the series mean	N_0 from the series 90% conf.
31	420.9	126 867	126 920	115 990	139 470 78 926
32	388.3	138 885	138 128	155 470	166 610 144 260
34	340.0	164 508	161 823	173 420	181 800 166 880

TAB. 1: NOMINAL SAFE LIFE VALUES CALCULATED BY DIFFERENT METHODS

WOHLER CURVE IMPROVEMENTS

Perfect modelling of the random process involved in air and ground fatigue loads is not possible by test. Even a flight record containing every detail of the loads imposed in their correct time and space orientation is still only a single sample taken from a potentially infinite series. Every safe service life prediction is based on some kind of cumulative damage calculation. The theoretical and practical problems involved in arriving at a perfect cumulative damage theory are outside the scope of this paper, but we should consider briefly a small but important detail amenable to improvement by the introduction of computer-based methods of calculation.

The classical form of handling fatigue data is the Wohler (S-N) curve. The current simplified form of plot showing part of the 50 percent failure probability data as two straight lines on semi-log paper is wholly inadequate and out-dated when compared with the best statistical methods for evaluation of single level tests — that is, Weibull.

While several realistic smoothing functions have been proposed for smoothing S-N curves (Ref. 10), even the best of these are limited in their application because they are confined to the 50 percent failure probability level. In only a few cases has any attempt been made to improve this situation. Figure 7 shows one made by the author. However, this may not be a particularly good solution, as cross-plotting between the different stress levels is at present done only by eye.

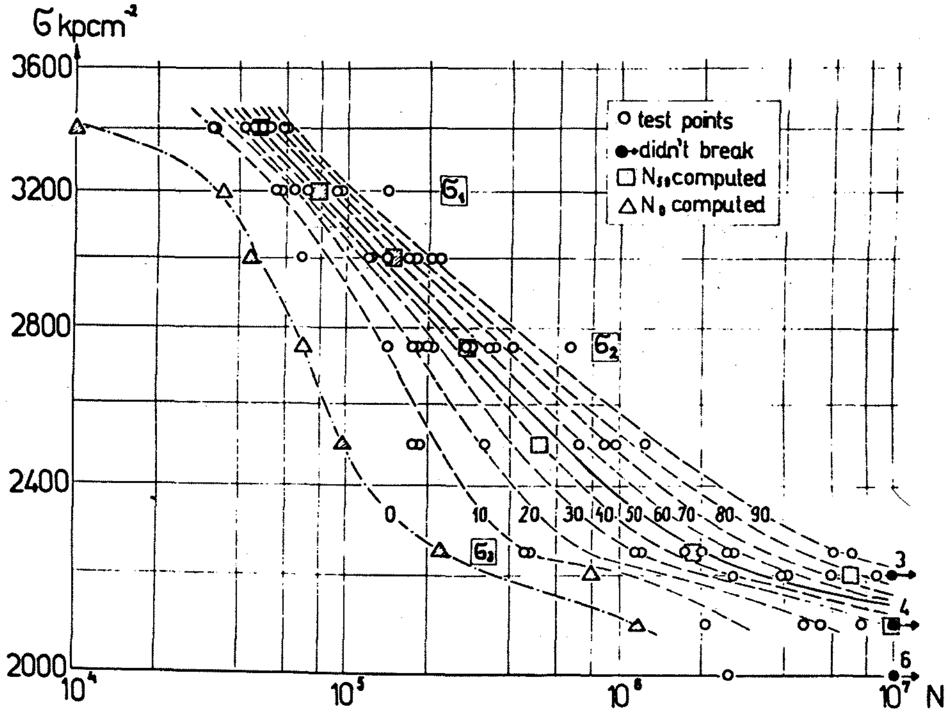

FIG. 7: WOHLER-CURVE SYSTEM FOR ROTATING-BENDING TESTS (REF. 5)

We therefore plan to conduct some trials using an analytical cross-plot method at all probability levels. The problem is generally similar to the experimental parameter case as the S-N curve descriptive function may be written as:

$$P = P(\sigma, N) \qquad (10)$$

However, it would be incorrect simply to copy the evaluation method outlined. It is known from theory as well as from experience that the safe life ration N_0/N_{50} is strongly dependent upon the stress level. We therefore intend to start our investigations by development of a parametric curve system from one of the 50 percent formulae, and it is hoped that this may lead to the development of a higher order unified evaluation method. However, we are at the moment handicapped by a lack of sufficient reliable experimental data.

CHARPY IMPACT TESTS

Charpy impact tests have one thing in common with fatigue tests, that is, the problem of substantial scatter in test results. Material properties are incompletely defined by the mean of the test results, and for adequate safety extrapolation by a suitable distribution function to a very low nominal failure probability is necessary.

The methods applied are similar to the fatigue evaluation process, and the results we have obtained up to the present time are summarized in the author's original paper.

CONCLUSIONS

Reliable prediction of low failure probability fatigue lives is possible only by extrapolation using lower-bounded statistical distribution functions. Mathematical difficulties in calculation of confidence limits using the three-parameter Weibull distribution may be avoided by the use of discrete probability variables.

In the field of experimental development directed towards fatigue life improvement, the proposed unified test evaluation procedure can give substantial savings in the number of test specimens required, and further developments of this procedure may lead to successful analytical cross-plotting between different stress levels at all failure probability levels.

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